DCS/CSCI 2350: Social & Economic Networks

Matching

Reading: Ch. 10 of EK & Handout for stable marriage

Mohammad T. Irfan

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60 Lives, 30 Kidneys, All Linked

By KEVIN SACK FEB. 18, 2012

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RIVERSIDE, Calif. — Rick Ruzzamenti admits to being a tad impulsive. He traded his Catholicism for Buddhism in a revelatory flash. He married a Vietnamese woman he had only just met. And then a year ago, he decided in an instant to donate his left kidney to a stranger.

In February 2011, the desk clerk at Mr. Ruzzamenti's yoga studio told him she had recently donated a kidney to an ailing friend she had bumped into at Target. Mr. Ruzzamenti, 44, had never even donated blood, but the story so captivated him that two days later he called Riverside Community Hospital to ask how he might do the same thing.

FROM START TO FINISH A donation by a Good Samaritan, Rick Ruzzamenti, upper left, set in motion a 60-person chain of transplants that ended with a kidney for Donald C. Terry Jr., bottom right.

Halfway across the country, in Joliet, Ill., Donald C. Terry Jr. needed a kidney in the worst way. Since receiving a diagnosis of diabetes-related renal disease in his mid-40s, he had endured the burning and bloating and dismal tedium of dialysis for nearly a year. With nobody in his family willing or able to give him a kidney, his doctors warned that it might take five years to crawl up the waiting list for an organ from a deceased donor.

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A Record Chain of Kidney Transplants
FEB. 18, 2012

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Alvin Roth
Nobel Prize 2012

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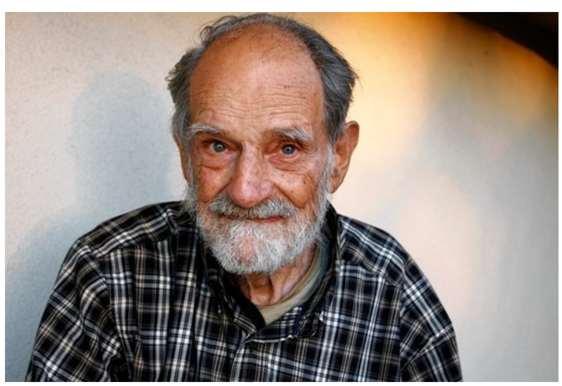

Lloyd Shapley
Nobel Prize 2012

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Stable matching

Given n men and n women, where each man ranks all women and each woman ranks all men, find a stable matching.

- Stable matching: no pair X and Y (not matched to each other) who prefer each other over their matched partners.
  - Such X & Y: "blocking pair"

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Perfect matching

Each person is matched to another

Necessary condition: # men = # women

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Demo

https://gale-shapley.com

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Is there always a perfect matching?

- Yes, Gale-Shapley algorithm (1962)
- *Deferred acceptance algorithm*

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Gale-Shapley algorithm: Men-proposing version

```
assign each person to be free ;
while some man m is free do
begin
    w := first woman on m's list to whom m has not yet proposed ;
    if w is free then
        assign m and w to be engaged {to each other}
    else
        if w prefers m to her fiancé m' then
            assign m and w to be engaged and m' to be free
        else
            w rejects m {and m remains free}
end ;
output the stable matching consisting of the n engaged pairs
```

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Gale-Shapley algorithm

- Thm 1.2.1. The algorithm terminates with a stable matching.
- Thm 1.2.2. Men-proposing version is men-optimal [ordering of men doesn't matter]
- Thm 1.2.3. Men proposing version is the worst for women [each woman gets the worst man <u>subject to the matching being stable</u>]

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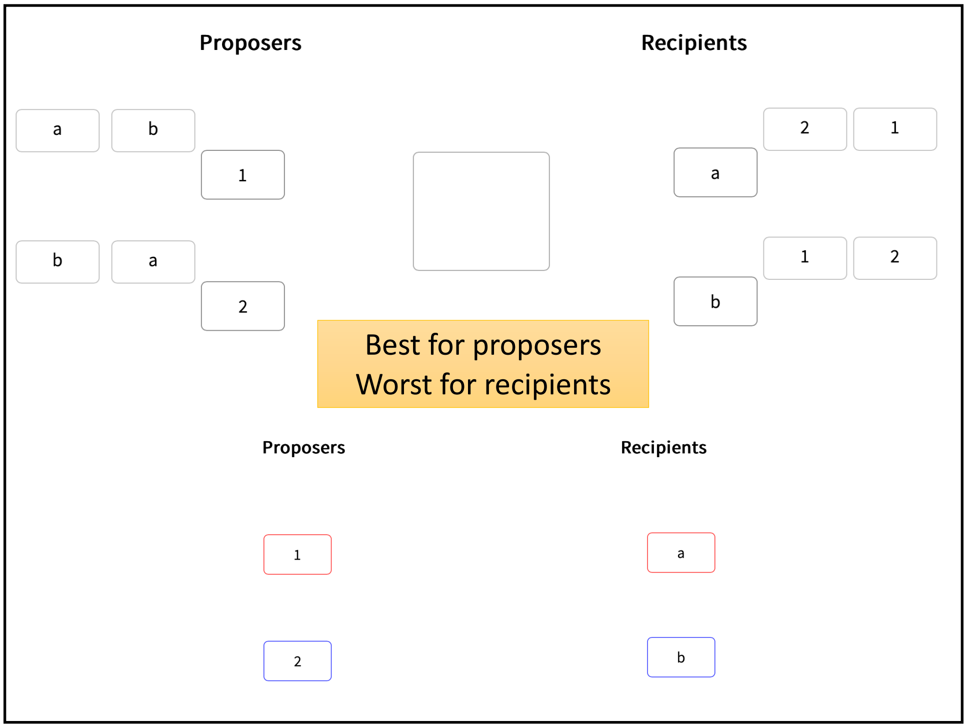
Proposers
Recipients
a
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Best for proposers
Worst for recipients
Proposers
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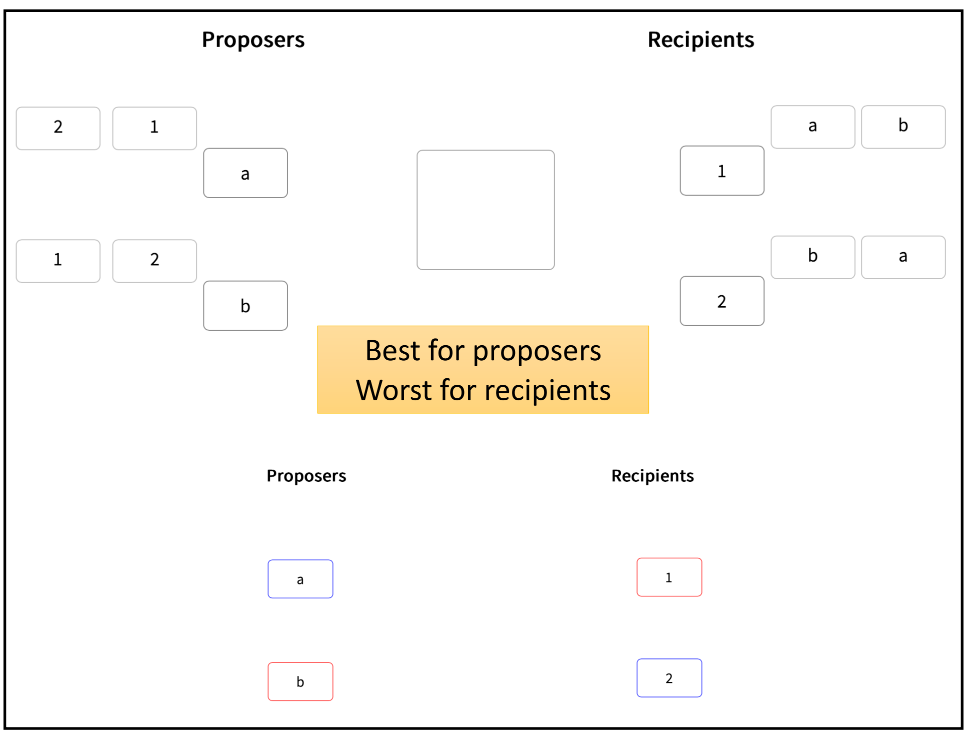
Proposers
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Best for proposers
Worst for recipients
Proposers
Recipients
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Applications

beyond kidney exchange

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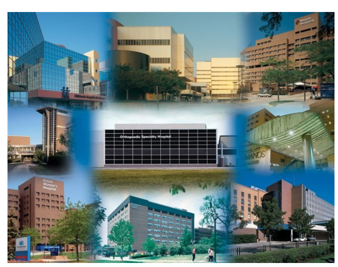
Residency matching

Hospitals interview candidates and rank them



Candidates rank hospitals that interviewed them

THE MATCH
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SECTIONS HOME SEARCH The New York Times

N.Y. / REGION

How Game Theory Helped Improve New York City's High School Application Process

By TRACY TULLIS DEC. 5, 2014

Tuesday was the deadline for eighth graders in New York City to submit applications to secure a spot at one of 426 public high schools. After months of school tours and tests, auditions and interviews, 75,000 students have entrusted their choices to a computer program that will arrange their school assignments for the coming year. The weeks of research and deliberation will be reduced to a fraction of a second of mathematical calculation: In just a couple of hours, all the sorting for the Class of 2019 will be finished.

To middle-school students and their parents, the high-school admissions process is a grueling and universally loathed rite of passage. But as awful as it can be, it used to be much worse. In the late 1990s, for instance, tens of thousands of children were shunted off to schools that had nothing going for them, it seemed, beyond empty desks. The process was so byzantine it appeared nothing short of a Nobel Prize-worthy algorithm could fix it.

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NYC high school matching

- Around 80K 8-th graders are matched to around 500 high schools
- Each student ranks at most 12 schools
- Schools rank applicants
  - 'But schools continue to tell parents and students — "with a wink" — that they may be penalized if they don't list their school first.' (https://www.dnainfo.com/new-york/20161115/kensington/nyc-high-school-admissions-ranking)
- Match by DOE

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Content delivery networks (CDN)

Algorithmic Nuggets in Content Delivery

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The authors take full responsibility for this article's technical content. Comments can be posted through CCR Online.

ABSTRACT

This paper "peeks under the covers" at the subsystems that provide the basic functionality of a leading content delivery network. Based on our experiences in building one of the largest distributed systems in the world, we illustrate how sophisticated algorithmic research has been adapted to balance the load between and within server clusters, manage the caches on servers, select paths through an overlay routing network, and elect leaders in various contexts. In each instance, we first explain the theory underlying the algorithms, then introduce practical considerations not captured by the theoretical models, and finally describe what is implemented in practice. Through these examples, we highlight the role of algorithmic research in the design of complex networked systems. The paper also illustrates the close synergy that exists between research and industry where research ideas cross over into products and product requirements drive future research.

1. INTRODUCTION

The top-three objectives for the designers and operators of a content delivery network (CDN) are high reliability, fast and consistent performance, and low operating cost.

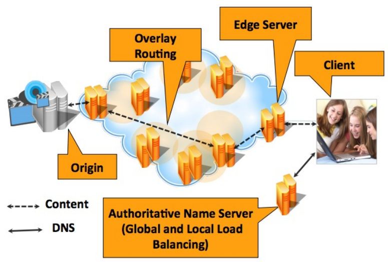

Figure 1: A CDN serves content in response to a client's request.

CDN's authoritative name server. The authoritative name

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Andrew M. Daniels

Actions

To: Mohammad Irfan

Inbox

Wednesday, June 25, 2014 11:06 PM

- You replied on 6/25/2014 11:43 PM.

Hi Professor,

I hope you're having a good summer! Things are going well at Akamai - I'm building an engine in Python to ping servers over different protocols, from which I'm collecting data to analyze.

I thought you might be interested in an application of the stable marriage algorithm (from class) that's used at Akamai. Essentially, they use an adapted version of the algorithm to match groups of end users with their servers for content delivery. Pretty cool seeing this come up!

- Andrew

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Matching market

Starter model: Buyers mark goods acceptable or not

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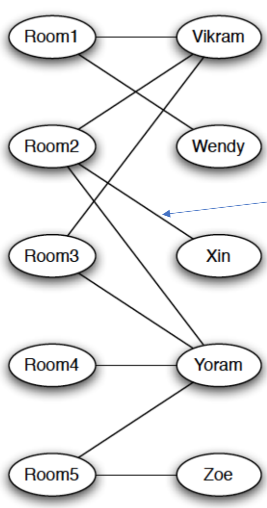
Bipartite matching problem

Find a "perfect matching" in a bipartite graph with equal number of nodes in each side



Each edge: The room is "acceptable" by the student

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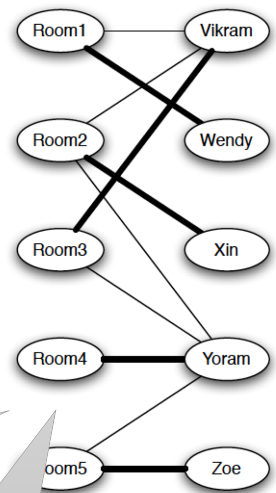
Perfect matching



Choice of edges in a bipartite graph such that each node is the endpoint of exactly one of the chosen edges.

- Interpretation?

Dark edges are the chosen edges—also known as the **assignment**

Difference between bipartite matching and stable marriage?
(There also, we wanted a perfect matching.)

Can you change the graph so that there exists no perfect matching?

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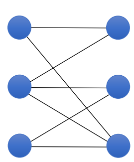
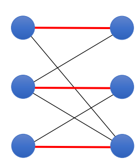
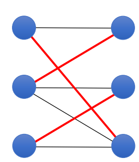
Perfect matching: more examples



A bipartite graph



One perfect matching



Another perfect matching

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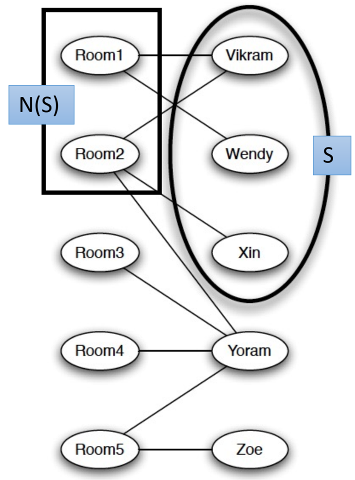
Constricted set

A set of nodes S is constricted if its neighbor set N(S) has less number of nodes than S

$|N(S)| < |S|$

(Note: We deleted the edge between Room3—Vikram from the previous example.)



(b) *A constricted set demonstrating there is no perfect matching*

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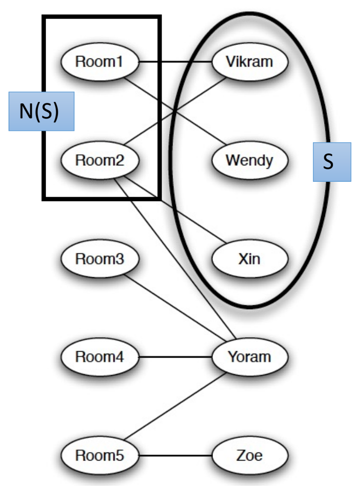
Implications

Constricted set ➔
Perfect matching is impossible

Reverse is also true!



(b) *A constricted set demonstrating there is no perfect matching*

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Matching Theorem/Hall's Theorem
Konig (1931), Hall (1935)

A bipartite graph with equal numbers of nodes on the left and right has

no perfect matching *if and only if* there's a constricted set

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But not all dorm rooms are same...
Model with valuations

- Each student has a valuation for each room
- Find a perfect matching that maximizes the sum of the valuations
- Social welfare = sum of the valuations in a matching

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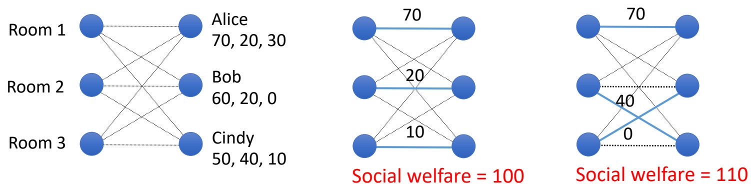
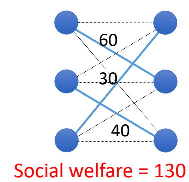
Model with valuations

- Many different perfect matchings:



How to find a perfect matching that maximizes the social welfare?

→ Optimal assignment



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More general matching markets

Valuations and optimal assignment

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Model

- n sellers, each is selling a house
  - Each house has a **price**
- n buyers
  - Each buyer has a **valuation** for each house
  - Buyer's **payoff** = valuation – price

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- Assumption: buyers are not stupid
  - Maximize their payoffs
  - Maximum payoff must also be >= 0
- Preferred seller graph
  - Bipartite graph between buyers and sellers where every edge encodes a buyer's maximum payoff (>= 0)

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Observations

- When buyers maximize valuation – price: prices determine perfect matching
- Price of a house too low ➔ ?
- Price too high ➔ ?

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What we want

- Determine the “right” price to get a perfect matching in the preferred seller graph
- **Market clearing prices (MCP):** The set of prices at which we get a perfect matching
- It would be awesome if the perfect matching is also an **optimal assignment!**
  - Maximizes social welfare (i.e., sum of the buyers’ valuations in that assignment)

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Good news

- Any MCP gives an optimal assignment
  - That is, any MCP maximizes social welfare
- Does an MCP (the "right" price) always exist?
  - Constructive proof (by an algorithm)

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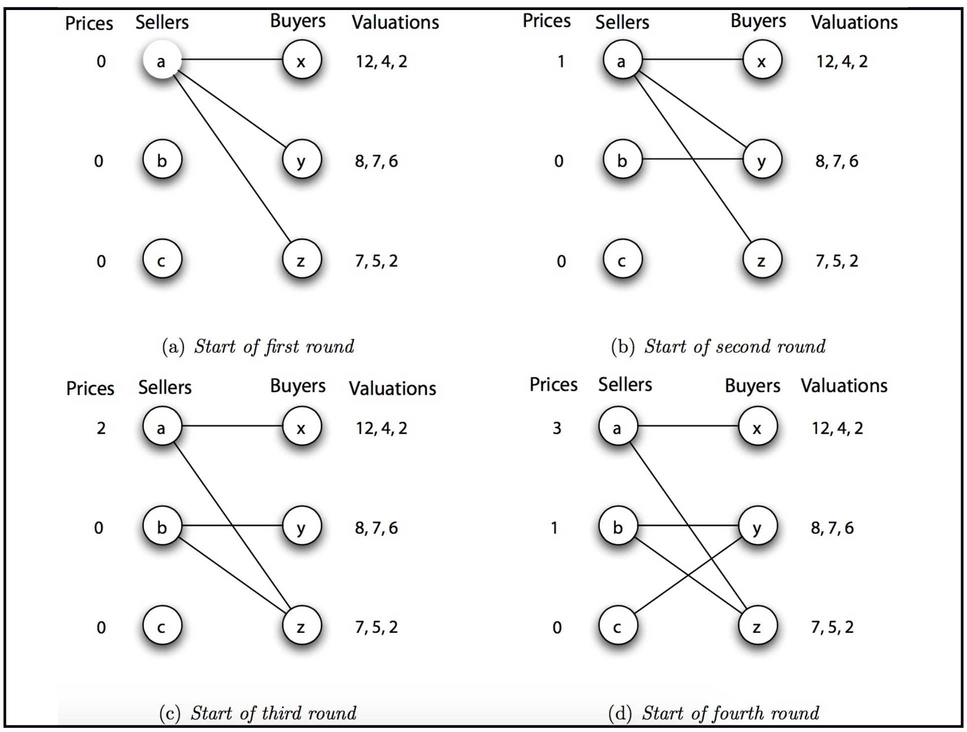

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Algorithm for MCP

Goal: compute MCP: prices for which there exists a perfect matching in the preferred seller graph

Algorithm

1. Initialize prices to 0
2. Buyers react by choosing their preferred seller(s)
3. If resulting graph has a perfect matching then done! Otherwise, find any constricted set, and increase the price of its neighbors by 1;
   (Normalize the prices—by decreasing **all** prices by the same amount so that at least one price is 0);
   Go to step 2

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Theorem: MCP maximizes each buyer's payoff as well as the social welfare

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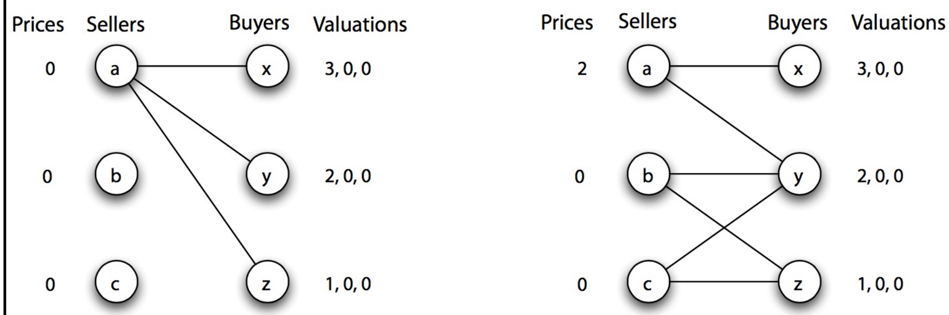
2nd price auction

Single-item auction is a matching market!



(a) *Start of the Auction*

(b) *End of the Auction*

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